

# Graphical amplitudes from SCET

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We discuss the relationship between the graphical amplitudes  $T$ ,  $C$ ,  $P$  ... used to parameterize nonleptonic  $B$  decay amplitudes, and matrix elements of operators in the soft-collinear effective theory (SCET) at leading order in  $\Lambda/m_b$ . Using the SU(3) flavor symmetry of the weak Hamiltonian we derive all-order constraints on the Wilson coefficients of SCET operators.

There are several theoretical approaches to treating exclusive non-leptonic  $B$  decays to two or more mesons. A first class of approaches uses flavor symmetries of QCD, in particular SU(3) [1, 2]. One then decomposes all possible amplitudes into the general set of reduced matrix elements in SU(3), and all relative factors can then be obtained from group theory. An alternative approach is the use of so called graphical amplitudes, which are defined by the topology of the quarks in a given diagram [3, 4, 5]. It was shown that there exists an equivalence between the graphical amplitudes method and the SU(3) reduced matrix elements [1], however a complete classification of all possible Wick contractions is rather complex [6]. The main complication is due to the appearance of rescattering effects [7]. For example, the color-suppressed decay  $\bar{B}^0 \rightarrow D^0 \pi^0$  can proceed through an intermediate color-allowed hadronic channel  $\bar{B}^0 \rightarrow D^+ \pi^-$ , followed by strong rescattering. In many analyses dynamical assumption about the sizes of the various graphical amplitudes are made, and certain graphical amplitudes are omitted.

A second class of approaches goes beyond using flavor symmetries and uses the limit  $m_b \rightarrow \infty$  to simplify the amplitudes. This was started with the perturbative QCD [8] method and the QCD factorization approach [9], and recently an effective theory treatment of these decays in the framework of soft collinear effective theory (SCET) [10] was developed [12, 13]. In this letter we show that there is a simple relationship between the graphical amplitudes and matrix elements of SCET operators, and we will prove the dynamical assumptions usually made. The use of isospin and SU(3) symmetry in SCET is also discussed.

The electroweak Hamiltonian mediating non-leptonic  $B$  decays is given by the  $\Delta B = 1$  Hamiltonian, which reads

$$H_W = \frac{G_F}{\sqrt{2}} \left[ \sum_{p=u,c} \lambda_p (C_1 O_1^p + C_2 O_2^p) - \lambda_t \sum_{i=3}^{10} C_i O_i \right],$$

where the CKM factor is  $\lambda_p = V_{pb} V_{pf}^*$  with  $f = d, s$  for  $\Delta S = 0, 1$  transitions, respectively. The operators  $O_{1,2}$  are the well known current-current operators

$$O_1^p = (\bar{p}b)_{V-A} (\bar{f}p)_{V-A}, \quad O_2^p = (\bar{p}_\beta b_\alpha)_{V-A} (\bar{f}_\alpha p_\beta)_{V-A},$$

the operators  $O_{3-6}$  are known as strong penguin opera-

tors

$$O_{3,4} = \{ (\bar{f}b)_{V-A} (\bar{q}q)_{V-A}, (\bar{f}_\beta b_\alpha)_{V-A} (\bar{q}_\alpha q_\beta)_{V-A} \},$$

$$O_{5,6} = \{ (\bar{f}b)_{V-A} (\bar{q}q)_{V+A}, (\bar{f}_\beta b_\alpha)_{V-A} (\bar{q}_\alpha q_\beta)_{V+A} \},$$

and the remaining operators denote electroweak penguin operators, whose definition can be obtained from Ref. [14].

The graphical amplitudes are defined as follows [3, 4, 5]

- *Tree amplitude  $T$ :*  
Operators  $O_{1,2}^u$  with the  $f$  quark and the  $\bar{u}$  quark of operator forming a meson.
- *Color suppressed amplitude  $C$ :*  
Operators  $O_{1,2}^u$  with the  $f$  quark and the spectator quark of the  $B$  forming a meson.
- *Penguin amplitudes  $P^u, P^c$ , or  $P^t$ :*  
Operators  $O_{1,2}^u, O_{1,2}^c$  with the two identical light quarks forming a virtual loop, or contributions from  $O_{3-6}$ , where in all three cases the  $f$  quark ends up in one of the light mesons.
- *Annihilation amplitude  $A$ :*  
Operators  $O_{1,2}^u$  with the  $u$  and  $\bar{b}$  quark of operator forming the  $B$  meson.
- *Exchange amplitude  $E$ :*  
Operators  $O_{1,2}^u$  with the  $f$  and  $\bar{b}$  quark of operator forming the  $B$  meson.
- *Penguin annihilation amplitude  $PA^{u,c,t}$ :*  
Operators  $O_{1,2}^u, O_{1,2}^c$  with the two identical light quarks forming a virtual loop or contributions from  $O_{3-6}$ , with the  $f$  quark ending up in the  $B$  mesons.
- *Electroweak penguin amplitudes  $EW^T$ :*  
Operators  $O_{7-10}$  with the  $f$  quark and the spectator quark of the  $B$  forming a meson.
- *Electroweak penguin amplitudes  $EW^C$ :*  
Operators  $O_{7-10}$  with the  $f$  quark and the  $\bar{q}$  quark forming a meson.
- *Electroweak penguin amplitudes  $EW^P$ :*  
Operators  $O_{7-10}$  with the  $f$  quark and the  $\bar{q}$  quark forming a virtual loop.
- *Electroweak penguin amplitudes  $EW^{A,E,PA}$ :*  
Operators  $O_{7-10}$  with the topologies identical to the  $A, E, PA$  topologies defined above.

All amplitudes in  $B \rightarrow M_1 M_2$  decays (with  $M_i$  being any light meson) can be written in terms of these graphical amplitudes. These relations are given in tabular form in Refs. [4, 5]. The SU(3) analysis in terms of graphical amplitudes is often supplemented with dynamical assumptions about their sizes, and some graphical amplitudes are typically neglected.

In SCET one uses the fact that the two light mesons have large energy much larger than  $\Lambda_{\text{QCD}}$ . This allows to match the Hamiltonian given in Eq. (1) at the scale  $m_b$  onto the effective theory Hamiltonian

$$H_W = \frac{2G_F}{\sqrt{2}} \sum_{n, \bar{n}} \left\{ \sum_i \int [d\omega_j]_{j=1}^3 c_i(\omega_j) Q_i^{(0)}(\omega_j) + \sum_i \int [d\omega_j]_{j=1}^4 b_i(\omega_j) Q_i^{(1)}(\omega_j) + \mathcal{Q}_{c\bar{c}} + \dots \right\}, \quad (1)$$

where  $c_i^{(f)}(\omega_j)$  and  $b_i^{(f)}(\omega_j)$  are Wilson coefficients, the ellipses denote color-octet operators which do not contribute at leading order and higher order terms in  $\Lambda_{\text{QCD}}/Q$ ,  $Q = \{m_b, E\}$ , and  $\mathcal{Q}_{c\bar{c}}$  denotes operators containing a  $c\bar{c}$  pair. In the remainder of this paper we often omit the dependence of the Wilson coefficients on the  $\omega_j$ . In Eq. (1) the  $\mathcal{O}(\lambda^0)$  operators are [sum over  $q = u, d, s$ ]

$$\begin{aligned} Q_1^{(0)} &= [\bar{u}_{n, \omega_1} \not{n} P_L b_v] [\bar{f}_{\bar{n}, \omega_2} \not{n} P_L u_{\bar{n}, \omega_3}], \\ Q_{2,3}^{(0)} &= [\bar{f}_{n, \omega_1} \not{n} P_L b_v] [\bar{u}_{\bar{n}, \omega_2} \not{n} P_L R u_{\bar{n}, \omega_3}], \\ Q_4^{(0)} &= [\bar{q}_{n, \omega_1} \not{n} P_L b_v] [\bar{f}_{\bar{n}, \omega_2} \not{n} P_L q_{\bar{n}, \omega_3}], \end{aligned} \quad (2)$$

where we have omitted operators which give rise to flavor-singlet light mesons. Note that the operator  $Q_3$  is only required if electroweak penguins are included. The effective theory operators contain collinear fields along both  $n$  and  $\bar{n}$  directions [11]. Also required are operators suppressed by one power of the SCET expansion parameter, as explained in [13]. They are given by

$$\begin{aligned} Q_1^{(1)} &= \frac{-2}{m_b} [\bar{u}_{n, \omega_1} i g \not{n} \mathcal{B}_{n, \omega_4}^\perp P_L b_v] [\bar{f}_{\bar{n}, \omega_2} \not{n} P_L u_{\bar{n}, \omega_3}], \\ Q_{2,3}^{(1)} &= \frac{-2}{m_b} [\bar{f}_{n, \omega_1} i g \not{n} \mathcal{B}_{n, \omega_4}^\perp P_L b_v] [\bar{u}_{\bar{n}, \omega_2} \not{n} P_L R u_{\bar{n}, \omega_3}], \\ Q_4^{(1)} &= \frac{-2}{m_b} [\bar{q}_{n, \omega_1} i g \not{n} \mathcal{B}_{n, \omega_4}^\perp P_L b_v] [\bar{f}_{\bar{n}, \omega_2} \not{n} P_L q_{\bar{n}, \omega_3}]. \end{aligned} \quad (3)$$

The factorization properties of SCET can be used to simplify the matrix elements of these operators significantly, leading to a final expression for the amplitude of an arbitrary  $B \rightarrow M_1 M_2$  decay [13]

$$A = N_0 \left\{ f_{M_1} \int_0^1 du dz T_{1J}(u, z) \zeta_J^{BM_2}(z) \phi^{M_1}(u) + f_{M_1} \zeta^{BM_2} \int_0^1 du T_{1\zeta}(u) \phi^{M_1}(u) \right\} + \left\{ 1 \leftrightarrow 2 \right\} + \lambda_c^{(f)} A_{c\bar{c}}^{M_1 M_2}. \quad (4)$$

Here  $A_{c\bar{c}}^{M_1 M_2} = \langle M_1 M_2 | \mathcal{Q}_{c\bar{c}} | B \rangle$ ,  $N_0 = \frac{G_F m_B^2}{\sqrt{2}}$ , the  $f_M$ 's are decay constants of the meson  $M$  and the  $\zeta^{BM}$  and

$\zeta_J^{BM}(z)$  are transition matrix elements between the initial  $B$  meson and a final meson  $M$ . They are of the same order in the power counting and are identical to the non-perturbative parameters which appear in  $B \rightarrow M$  form factors [15].

The Wilson coefficients of the operators  $Q_{1-4}^{(0,1)}$  are given at tree level in [13]. To separate the contributions of full theory current-current operators, QCD penguin operators and electroweak penguin operators to the SCET matching coefficients, it is convenient to write the Wilson coefficients of the SCET operators as

$$\begin{aligned} c_i &= \lambda_u c_{iu} + \lambda_t [c_{it}^p + c_{it}^{\text{ew}}] \\ b_i &= \lambda_u c_{iu} + \lambda_t^{(f)} [b_{it}^p + b_{it}^{\text{ew}}]. \end{aligned} \quad (5)$$

Here  $c_{it}^p, b_{it}^p$  denote the terms proportional to the full theory Wilson coefficients  $C_{3-6}$  of the QCD penguin operators, while  $c_{it}^{\text{ew}}, b_{it}^{\text{ew}}$  denote the terms proportional to the full theory Wilson coefficients  $C_{7-10}$  of the electroweak penguin operators. With these definitions, the tree level Wilson coefficients are

$$\begin{aligned} c_{1u} &= C_1 + \frac{C_2}{N_c}, & c_{1t}^{\text{ew}} &= -\frac{3}{2} \left( C_{10} + \frac{C_9}{N_c} \right) \\ c_{2u} &= C_2 + \frac{C_1}{N_c}, & c_{2t}^{\text{ew}} &= -\frac{3}{2} \left( C_9 + \frac{C_{10}}{N_c} \right) \\ c_{3t}^{\text{ew}} &= -\frac{3}{2} \left( C_7 + \frac{C_8}{N_c} \right) \\ c_{4t}^p &= -\left( C_4 + \frac{C_3}{N_c} \right), & c_{4t}^{\text{ew}} &= \frac{1}{2} \left( C_{10} + \frac{C_9}{N_c} \right), \\ c_{1t}^p &= c_{2t}^p = c_{3u} = c_{3t}^p = c_{4u} = 0 \end{aligned} \quad (6)$$

and

$$\begin{aligned} b_{1u} &= C_1 + \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_2}{N_c} \\ b_{1t}^{\text{ew}} &= -\frac{3}{2} \left[ C_{10} + \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_9}{N_c} \right] \\ b_{2u} &= C_2 + \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_1}{N_c} \\ b_{2t}^{\text{ew}} &= -\frac{3}{2} \left[ C_9 + \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_{10}}{N_c} \right] \\ b_{3t}^{\text{ew}} &= -\frac{3}{2} \left[ C_7 + \left( 1 - \frac{m_b}{\omega_2} \right) \frac{C_8}{N_c} \right] \\ b_{4t}^p &= -C_4 - \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_3}{N_c} \\ b_{4t}^{\text{ew}} &= \frac{1}{2} \left[ C_{10} + \left( 1 - \frac{m_b}{\omega_3} \right) \frac{C_9}{N_c} \right]. \\ b_{1t}^p &= b_{2t}^p = b_{3u} = b_{3t}^p = b_{4u} = 0 \end{aligned} \quad (7)$$

The Wilson coefficients  $c_{1t}^p, c_{2t}^p$  and  $b_{1t}^p, b_{2t}^p$  vanish to all orders in matching, since the penguin operators  $O_{3-6}$  transform as a **3** under flavor SU(3), while  $Q_{1,2}$  transform as a combination of **3**,  **$\bar{6}$** , **15**. A similar argument gives  $c_{4u} = b_{4u} = 0$ . Chirality invariance requires that  $c_{3u} = b_{3u} = 0$  to all orders.

The graphical amplitudes are defined by the contractions of the quark lines of the operator with the quarks in the mesons. In QCD, this separation is difficult to define in an unambiguous way in terms of matrix elements of operators. SCET however allows this distinction, since each meson consists of interpolating fields with well defined collinear directions. The  $B$  meson is described by an interpolating field of two soft quarks, while the two light mesons are defined by interpolating fields of two collinear quarks in different light-like directions. At leading order in SCET there are no couplings between soft and collinear particles, or between collinear particles in different directions. Since the four quarks in the SCET operators in Eqs. (2) and (3) are either soft or have a well defined collinear direction, each of them ends up in a particular meson. Thus, the graphical amplitudes can be defined as matrix elements of SCET operators.

The relation of the graphical amplitudes to SCET matrix elements can be obtained without making use of any flavor symmetries. While this will result in too many graphical amplitudes to have any predictive power, it will illustrate the identification between graphical amplitudes and SCET matrix elements. We will later show how flavor symmetries can be used to reduce the set of amplitudes. We add subscripts  $M_1 M_2$  to any graphical amplitude to denote the final state of the decay. A general amplitude defined as  $A_{M_1 M_2} \equiv A(B \rightarrow M_1 M_2)$  is then given by

$$\begin{aligned} A_{M_1 M_2} = & \lambda_u (T_{M_1 M_2} + C_{M_1 M_2} + A_{M_1 M_2} + E_{M_1 M_2} \\ & + P_{M_1 M_2}^u + PA_{M_1 M_2}^u) \\ & + \lambda_c (P_{M_1 M_2}^c + PA_{M_1 M_2}^c) \\ & + \lambda_t (P_{M_1 M_2}^t + PA_{M_1 M_2}^t + EW_{M_1 M_2}). \end{aligned} \quad (8)$$

The graphical amplitudes are defined as before, but depend on the particular final state. Here  $EW$  denotes the sum of all possible electroweak penguin amplitudes  $EW^{T,C,P,A,E,PA}$ .

So far the matching contributions from charming penguins have not been fully worked out in SCET, thus their effects are not included in the SCET operators  $Q_{1-4}^{(0,1)}$ . Thus, we make no attempt here to derive explicit results for  $P_{M_1 M_2}^c$  and  $PA_{M_1 M_2}^c$  and simply leave them as unknown parameters

$$P_{M_1 M_2}^c = A_{ccM_1 M_2}^P, \quad PA_{M_1 M_2}^c = A_{ccM_1 M_2}^{PA}. \quad (9)$$

In  $B \rightarrow \pi\pi$  decays the charming penguins appear always in the combination  $A_{cc\pi\pi}^P + A_{cc\pi\pi}^{PA} = A_{cc}^{\pi\pi}$  where  $A_{cc}^{\pi\pi}$  is the amplitude introduced in Ref. [13]. However, in  $B \rightarrow K\pi$  only the  $A_{ccK\pi}^P$  amplitude contributes. We will neglect here charming penguin effects arising from electroweak penguins due to their small Wilson coefficients.

Next, consider the tree amplitudes  $T$  and  $C$ . Since they originate from the current-current operators, and do not contain a penguin contraction, the operators  $Q_4^{(0,1)}$  do not contribute. Furthermore, since the current-current operators are multiplied by  $\lambda_u$ , only the Wilson coefficients  $c_{1u-3u}$  and  $b_{1u-3u}$  are required. Finally, since the

$$B \rightarrow \pi\pi$$

| mode         | $T_{\pi\pi}$          | $C_{\pi\pi}$          | $P_{\pi\pi}^{u,c,t}$ | $PA_{\pi\pi}^c$ | $EW_{\pi\pi}^T$      | $EW_{\pi\pi}^C$       | $EW_{\pi\pi}^P$ |
|--------------|-----------------------|-----------------------|----------------------|-----------------|----------------------|-----------------------|-----------------|
| $\pi^+\pi^-$ | -1                    | 0                     | -1                   | -1              | 0                    | 1                     | -1              |
| $\pi^+\pi^0$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0                    | 0               | $\frac{1}{\sqrt{2}}$ | $\frac{3}{2\sqrt{2}}$ | 0               |
| $\pi^0\pi^0$ | 0                     | -1                    | 1                    | 1               | 1                    | $\frac{1}{2}$         | 1               |

TABLE I: SU(2) relations for the decays  $B \rightarrow \pi\pi$

$$B \rightarrow K\pi$$

| mode             | $T_{K\pi}$            | $C_{K\pi}$            | $P_{K\pi}^{u,c,t}$    | $PA_{K\pi}^c$ | $EW_{K\pi}^T$        | $EW_{K\pi}^C$         | $EW_{K\pi}^P$         |
|------------------|-----------------------|-----------------------|-----------------------|---------------|----------------------|-----------------------|-----------------------|
| $K^-\pi^+$       | -1                    | 0                     | -1                    | 0             | 0                    | 1                     | -1                    |
| $K^-\pi^0$       | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0             | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  | $-\frac{1}{\sqrt{2}}$ |
| $\bar{K}^0\pi^-$ | 0                     | 0                     | 1                     | 0             | 0                    | $\frac{1}{2}$         | 1                     |
| $\bar{K}^0\pi^0$ | 0                     | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  | 0             | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  |

TABLE II: SU(2) relations for the decays  $B \rightarrow K\pi$

$$B \rightarrow K\bar{K}$$

| mode           | $T_{K\bar{K}}$ | $C_{K\bar{K}}$ | $P_{K\bar{K}}^{u,c,t}$ | $PA_{K\bar{K}}^c$ | $EW_{K\bar{K}}^T$ | $EW_{K\bar{K}}^C$ | $EW_{K\bar{K}}^P$ |
|----------------|----------------|----------------|------------------------|-------------------|-------------------|-------------------|-------------------|
| $K^+K^-$       | 0              | 0              | 0                      | -1                | 0                 | 0                 | 0                 |
| $K^-K^0$       | 0              | 0              | 1                      | 0                 | 0                 | $\frac{1}{2}$     | 1                 |
| $\bar{K}^0K^0$ | 0              | 0              | 1                      | 1                 | 0                 | $\frac{1}{2}$     | 1                 |

TABLE III: SU(2) relations for the decays  $B \rightarrow K\bar{K}$

operator  $Q_3$  only receives contributions from electroweak penguin operators in the full theory,  $c_{3u} = b_{3u} = 0$ . The distinction between the  $T$  and  $C$  amplitudes is whether the quark  $f$  forms a light meson with a  $\bar{u}$  quark of the operator or with the spectator of the  $B$  meson. Since the two collinear quarks in the  $\bar{n}$  direction form one meson, while the collinear quark in the  $n$  direction combines with the spectator of the  $B$  in the second step of matching, the contributions to  $T$  are only from operators  $Q_1^{(0,1)}$  and those to  $C$  are from  $Q_2^{(0,1)}$ . This gives

$$\begin{aligned} T_{M_1 M_2} &= N_0 \langle M_1 M_2 | c_{1u} Q_1^{(0)} + b_{1u} Q_1^{(1)} | B \rangle \\ C_{M_1 M_2} &= N_0 \langle M_1 M_2 | c_{2u} Q_2^{(0)} + b_{2u} Q_2^{(1)} | B \rangle. \end{aligned} \quad (10)$$

In writing the matrix element of  $Q_i^{(0,1)}$  it is understood that the definition of the matrix elements involves time-ordered products with subleading interactions in SCET which couple a soft to a collinear quark. For details see [13, 15].

The other graphical amplitudes can be obtained in the same way. Since the three light quarks of the operators in SCET are all collinear, they can not be part of the  $B$  meson. Thus, to leading order in SCET we find

$$A_{M_1 M_2} = E_{M_1 M_2} = PA_{M_1 M_2}^{u,t} = 0. \quad (11)$$

The QCD penguin contributions are from matrix elements of the operator  $Q_4^{(0,1)}$  and we find

$$\begin{aligned} P_{M_1 M_2}^u &= N_0 \langle M_1 M_2 | c_{4u} Q_4^{(0)} + b_{4u} Q_4^{(1)} | B \rangle \\ P_{M_1 M_2}^t &= N_0 \langle M_1 M_2 | c_{4t}^P Q_4^{(0)} + b_{4t}^P Q_4^{(1)} | B \rangle. \end{aligned} \quad (12)$$

Finally, we need the electroweak penguin amplitudes. We find

$$\begin{aligned}
EW_{M_1 M_2}^T &= N_0 \langle M_1 M_2 | \sum_{j=2,3} c_{jt}^{\text{ew}} Q_j^{(0)} + b_{jt}^{\text{ew}} Q_j^{(1)} | B \rangle \\
EW_{M_1 M_2}^C &= N_0 \langle M_1 M_2 | (c_{1t}^{\text{ew}}/3)(3Q_1^{(0)} - Q_4^{(0)}) \\
&\quad + (b_{1t}^{\text{ew}}/3)(3Q_1^{(1)} - Q_4^{(1)}) | B \rangle \\
EW_{M_1 M_2}^P &= N_0 \langle M_1 M_2 | (c_{4t}^{\text{ew}} + c_{1t}^{\text{ew}}/3)Q_4^{(0)} \\
&\quad + (b_{4t}^{\text{ew}} + b_{1t}^{\text{ew}}/3)Q_4^{(1)} | B \rangle, \quad (13)
\end{aligned}$$

and at leading order

$$EW_{M_1 M_2}^A = EW_{M_1 M_2}^E = EW_{M_1 M_2}^{PA} = 0. \quad (14)$$

So far our results for the graphical amplitudes have been general, and no symmetry assumption has been made. Since the number of graphical amplitudes is larger than the number of decay modes, there is no predictive power. We start by assuming isospin symmetry. This gives a reduction in the number of amplitudes, which are labeled by the isospin content of the final state  $M_1 M_2$ . For example, all modes with two pions in the final state have graphical amplitudes labeled by the isospin content  $\pi\pi$ , for  $K\pi$  final states we label them by  $K\pi$ , and so on. In Tables I-III we give the relative factors between the graphical amplitudes for a given final state to the graphical amplitudes with given isospin content. These tables agree with the results in Ref. [4], if one takes into account that the EWP graphical amplitudes  $EW^{T,C}$  are defined with a different normalization than in Ref. [4]. We use the same definitions for  $PA^c$  as in Refs. [4] and reproduce here the results for these amplitudes for completeness. The results for final states with two vector mesons are identical to those for two pseudoscalar mesons.

The amplitudes of given isospin content can be expressed in terms of SCET parameters. These relations have a similar form for  $PP$  and  $VV$  final states. For flavor content  $K\pi$  they are given by

$$\begin{aligned}
T_{K\pi} &= -N_0 f_K [\langle c_{1u} \rangle_K \zeta^{B\pi} + \langle b_{1u} \zeta_J^{B\pi} \rangle_K] \\
C_{K\pi} &= -N_0 f_\pi [\langle c_{2u} \rangle_\pi \zeta^{BK} + \langle b_{2u} \zeta_J^{BK} \rangle_\pi] \\
P_{K\pi}^u &= -N_0 f_K [\langle c_{4u} \rangle_K \zeta^{B\pi} + \langle b_{4u} \zeta_J^{B\pi} \rangle_K] \\
P_{K\pi}^t &= -N_0 f_K [\langle c_{4t}^p \rangle_K \zeta^{B\pi} + \langle b_{4t}^p \zeta_J^{B\pi} \rangle_K] \\
EW_{K\pi}^T &= N_0 f_\pi [\langle c_{2t}^{\text{ew}} - c_{3t}^{\text{ew}} \rangle_\pi \zeta^{BK} + \langle (b_{2t}^{\text{ew}} - b_{3t}^{\text{ew}}) \zeta_J^{BK} \rangle_\pi] \\
EW_{K\pi}^C &= \frac{2N_0}{3} f_K [\langle c_{1t}^{\text{ew}} \rangle_K \zeta^{B\pi} + \langle b_{1t}^{\text{ew}} \zeta_J^{B\pi} \rangle_K] \\
EW_{K\pi}^P &= -\frac{N_0}{3} f_K [\langle 3c_{4t}^{\text{ew}} + c_{1t}^{\text{ew}} \rangle_K \zeta^{B\pi} \\
&\quad + \langle (3b_{4t}^{\text{ew}} + b_{1t}^{\text{ew}}) \zeta_J^{B\pi} \rangle_K]. \quad (15)
\end{aligned}$$

The results for  $K^*\rho$  are the same with changed subscripts for the flavor content and an opposite sign between the two Wilson coefficients  $[(c_{2t}^{\text{ew}} + c_{3t}^{\text{ew}}), (b_{2t}^{\text{ew}} + b_{3t}^{\text{ew}})]$  in  $EW_{K\pi}^T$ . The other channels  $\pi\pi, K\bar{K}, \rho\rho, K^*\bar{K}^*$  can be obtained

from this by straightforward substitutions. For simplicity of notation we defined

$$\begin{aligned}
\langle c_{iu} \rangle_{M_1} &= \int_0^1 du c_{iu}(u) \phi_{M_1}(u), \\
\langle b_{iu} \zeta_J^{BM_2} \rangle_{M_1} &= \int_0^1 du dz b_{iu}(u, z) \phi_{M_1}(u) \zeta_J^{BM_2}(z). \quad (16)
\end{aligned}$$

The  $B \rightarrow VP$  decays can be described analogously. For this case Bose symmetry does not constrain the quantum numbers of the final state, which allows more independent amplitudes. We define them using the convention of [16] which adds one index  $P$  or  $V$  depending on the light meson which picks up the spectator in the  $B$  meson. Our results are presented in Tables IV-VIII. The graphical amplitudes can again be expressed in terms of decay constants and  $B \rightarrow M$  transition matrix elements. We find

$$\begin{aligned}
T_{PV}^P &= -N_0 f_P [\langle c_{1u} \rangle_V \zeta^{BP} + \langle b_{1u} \zeta_J^{BP} \rangle_V] \\
C_{PV}^P &= -N_0 f_V [\langle c_{2u} \rangle_V \zeta^{BP} + \langle b_{2u} \zeta_J^{BP} \rangle_V] \\
P_{PV}^u &= -N_0 f_V [\langle c_{4u} \rangle_V \zeta^{BP} + \langle b_{4u} \zeta_J^{BP} \rangle_V] \\
P_{PV}^t &= -N_0 f_V [\langle c_{4t}^p \rangle_V \zeta^{BP} + \langle b_{4t}^p \zeta_J^{BP} \rangle_V] \\
EW_{PV}^{TP} &= N_0 f_V [\langle c_{2t}^{\text{ew}} - c_{3t}^{\text{ew}} \rangle_V \zeta^{BP} + \langle (b_{2t}^{\text{ew}} - b_{3t}^{\text{ew}}) \zeta_J^{BP} \rangle_V] \\
EW_{PV}^{CP} &= \frac{2N_0}{3} f_V [\langle c_{1t}^{\text{ew}} \rangle_V \zeta^{BP} + \langle b_{1t}^{\text{ew}} \zeta_J^{BP} \rangle_V] \\
EW_{PV}^{PP} &= -\frac{N_0}{3} f_V [\langle 3c_{4t}^{\text{ew}} + c_{1t}^{\text{ew}} \rangle_V \zeta^{BP} \\
&\quad + \langle (3b_{4t}^{\text{ew}} + b_{1t}^{\text{ew}}) \zeta_J^{BP} \rangle_V], \quad (17)
\end{aligned}$$

where  $P$  and  $V$  denote any pseudoscalar or vector meson. The expressions for  $G_{PV}^V$ , where  $G = \{T, C, P, EW^T, EW^C, EW^P\}$  denotes any of the graphical amplitudes, can be obtained from Eq. (17) by taking  $P \leftrightarrow V$  on the right hand side and changing the sign between  $c_{2t}^{\text{ew}}$  and  $c_{3t}^{\text{ew}}$  in  $EW_{PV}^T$ .

The relations presented in Eqs. (15) and (17) are correct to all orders in the perturbative matching calculation. Currently, the complete set of required Wilson coefficients are only available at tree level, and we give the resulting relations at that order in perturbation theory. From Eq. (6) one can see that the Wilson coefficients  $c_i(\omega_j)$  are independent of the arguments at tree level and at that order we thus find the simple relations

$$\langle c_{iu} \rangle_M = c_{iu}, \quad \langle c_{it}^p \rangle_M = c_{it}^p, \quad \langle c_{it}^{\text{ew}} \rangle_M = c_{it}^{\text{ew}}. \quad (18)$$

The Wilson coefficients  $b_i(\omega_j)$ , given in Eq. (7), depend

$$\bar{B} \rightarrow \rho\pi$$

| mode          | $T_{\rho\pi}^P$       | $T_{\rho\pi}^V$       | $C_{\rho\pi}^P$       | $C_{\rho\pi}^V$       | $P_{\rho\pi}^{(u,c,t)P}$ | $P_{\rho\pi}^{(u,c,t)V}$ | $PA_{\rho\pi}^{cP}$ | $PA_{\rho\pi}^{cV}$ | $EW_{\rho\pi}^{TP}$  | $EW_{\rho\pi}^{TV}$  | $EW_{\rho\pi}^{CP}$   | $EW_{\rho\pi}^{CV}$   | $EW_{\rho\pi}^{PP}$   | $EW_{\rho\pi}^{PV}$   |
|---------------|-----------------------|-----------------------|-----------------------|-----------------------|--------------------------|--------------------------|---------------------|---------------------|----------------------|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\rho^+\pi^-$ | 0                     | -1                    | 0                     | 0                     | 0                        | -1                       | -1                  | -1                  | 0                    | 0                    | 0                     | 1                     | 0                     | -1                    |
| $\pi^+\rho^-$ | -1                    | 0                     | 0                     | 0                     | -1                       | 0                        | -1                  | -1                  | 0                    | 0                    | 1                     | 0                     | -1                    | 0                     |
| $\rho^0\pi^0$ | 0                     | 0                     | $-\frac{1}{2}$        | $-\frac{1}{2}$        | $\frac{1}{2}$            | $\frac{1}{2}$            | $\frac{1}{2}$       | $\frac{1}{2}$       | $\frac{1}{2}$        | $\frac{1}{2}$        | $\frac{1}{4}$         | $\frac{1}{4}$         | $\frac{1}{2}$         | $\frac{1}{2}$         |
| $\rho^-\pi^0$ | $-\frac{1}{\sqrt{2}}$ | 0                     | 0                     | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$    | $\frac{1}{\sqrt{2}}$     | 0                   | 0                   | 0                    | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  | $\frac{1}{2\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  |
| $\pi^-\rho^0$ | 0                     | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0                     | $\frac{1}{\sqrt{2}}$     | $-\frac{1}{\sqrt{2}}$    | 0                   | 0                   | $\frac{1}{\sqrt{2}}$ | 0                    | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  | $\frac{1}{\sqrt{2}}$  | $-\frac{1}{\sqrt{2}}$ |

TABLE IV: SU(2) relations for the decays  $B \rightarrow \rho\pi$ 

$$\bar{B} \rightarrow K\bar{K}^*$$

| mode              | $P_{K\bar{K}^*}^{(u,c,t)P}$ | $P_{K\bar{K}^*}^{(u,c,t)V}$ | $PA_{K\bar{K}^*}^{cP}$ | $PA_{K\bar{K}^*}^{cV}$ | $EW_{K\bar{K}^*}^{CP}$ | $EW_{K\bar{K}^*}^{CV}$ | $EW_{K\bar{K}^*}^{PP}$ | $EW_{K\bar{K}^*}^{PV}$ |
|-------------------|-----------------------------|-----------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $K^{*-}K^0$       | 0                           | 1                           | 0                      | 0                      | 0                      | $\frac{1}{2}$          | 0                      | 1                      |
| $\bar{K}^{*0}K^0$ | 0                           | 1                           | 1                      | 1                      | 0                      | $\frac{1}{2}$          | 0                      | 1                      |
| $K^{*-}K^+$       | 0                           | 0                           | -1                     | -1                     | 0                      | 0                      | 0                      | 0                      |

TABLE V: SU(2) relations for the decays  $\bar{B} \rightarrow K\bar{K}^*$ 

$$\bar{B} \rightarrow \bar{K}K^*$$

| mode              | $P_{\bar{K}K^*}^{(u,c,t)P}$ | $P_{\bar{K}K^*}^{(u,c,t)V}$ | $PA_{\bar{K}K^*}^{cP}$ | $PA_{\bar{K}K^*}^{cV}$ | $EW_{\bar{K}K^*}^{CP}$ | $EW_{\bar{K}K^*}^{CV}$ | $EW_{\bar{K}K^*}^{PP}$ | $EW_{\bar{K}K^*}^{PV}$ |
|-------------------|-----------------------------|-----------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $K^{*0}K^-$       | 1                           | 0                           | 0                      | 0                      | $\frac{1}{2}$          | 0                      | 1                      | 0                      |
| $K^{*0}\bar{K}^0$ | 1                           | 0                           | 1                      | 1                      | $\frac{1}{2}$          | 0                      | 1                      | 0                      |
| $K^{*+}K^-$       | 0                           | 0                           | -1                     | -1                     | 0                      | 0                      | 0                      | 0                      |

TABLE VI: SU(2) relations for the decays  $\bar{B} \rightarrow \bar{K}K^*$ 

$$\bar{B} \rightarrow K\rho$$

| mode              | $T_{K\rho}^P$ | $T_{K\rho}^V$         | $C_{K\rho}^P$         | $C_{K\rho}^V$ | $P_{K\rho}^{(u,c,t)P}$ | $P_{K\rho}^{(u,c,t)V}$ | $PA_{K\rho}^{cP}$ | $PA_{K\rho}^{cV}$ | $EW_{K\rho}^{TP}$    | $EW_{K\rho}^{TV}$ | $EW_{K\rho}^{CP}$ | $EW_{K\rho}^{CV}$     | $EW_{K\rho}^{PP}$ | $EW_{K\rho}^{PV}$     |
|-------------------|---------------|-----------------------|-----------------------|---------------|------------------------|------------------------|-------------------|-------------------|----------------------|-------------------|-------------------|-----------------------|-------------------|-----------------------|
| $K^-\rho^+$       | 0             | -1                    | 0                     | 0             | 0                      | -1                     | 0                 | 0                 | 0                    | 0                 | 0                 | 1                     | 0                 | -1                    |
| $K^-\rho^0$       | 0             | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0             | 0                      | $-\frac{1}{\sqrt{2}}$  | 0                 | 0                 | $\frac{1}{\sqrt{2}}$ | 0                 | 0                 | $\frac{1}{\sqrt{2}}$  | 0                 | $-\frac{1}{\sqrt{2}}$ |
| $\bar{K}^0\rho^-$ | 0             | 0                     | 0                     | 0             | 0                      | 1                      | 0                 | 0                 | 0                    | 0                 | 0                 | $\frac{1}{2}$         | 0                 | 1                     |
| $\bar{K}^0\rho^0$ | 0             | 0                     | $-\frac{1}{\sqrt{2}}$ | 0             | 0                      | $\frac{1}{\sqrt{2}}$   | 0                 | 0                 | $\frac{1}{\sqrt{2}}$ | 0                 | 0                 | $\frac{1}{2\sqrt{2}}$ | 0                 | $\frac{1}{\sqrt{2}}$  |

TABLE VII: SU(2) relations for the decays  $B \rightarrow K\rho$ 

$$\bar{B} \rightarrow K^*\pi$$

| mode                | $T_{K^*\pi}^P$        | $T_{K^*\pi}^V$ | $C_{K^*\pi}^P$ | $C_{K^*\pi}^V$        | $P_{K^*\pi}^{(u,c,t)P}$ | $P_{K^*\pi}^{(u,c,t)V}$ | $PA_{K^*\pi}^{cP}$ | $PA_{K^*\pi}^{cV}$ | $EW_{K^*\pi}^{TP}$ | $EW_{K^*\pi}^{TV}$   | $EW_{K^*\pi}^{CP}$    | $EW_{K^*\pi}^{CV}$ | $EW_{K^*\pi}^{PP}$    | $EW_{K^*\pi}^{PV}$ |
|---------------------|-----------------------|----------------|----------------|-----------------------|-------------------------|-------------------------|--------------------|--------------------|--------------------|----------------------|-----------------------|--------------------|-----------------------|--------------------|
| $K^{*-}\pi^+$       | -1                    | 0              | 0              | 0                     | -1                      | 0                       | 0                  | 0                  | 0                  | 0                    | 1                     | 0                  | -1                    | 0                  |
| $K^{*-}\pi^0$       | $-\frac{1}{\sqrt{2}}$ | 0              | 0              | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$   | 0                       | 0                  | 0                  | 0                  | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$  | 0                  | $-\frac{1}{\sqrt{2}}$ | 0                  |
| $\bar{K}^{*0}\pi^-$ | 0                     | 0              | 0              | 0                     | 1                       | 0                       | 0                  | 0                  | 0                  | 0                    | 1/2                   | 0                  | 1                     | 0                  |
| $\bar{K}^{*0}\pi^0$ | 0                     | 0              | 0              | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$    | 0                       | 0                  | 0                  | 0                  | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | 0                  | $\frac{1}{\sqrt{2}}$  | 0                  |

TABLE VIII: SU(2) relations for the decays  $B \rightarrow K^*\pi$ 

on one parameter at tree level. Thus we find

$$\langle b_{1u}\zeta_J^{BM_2}\rangle_{M_1} = \left[ C_1 + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_2}{N_c} \right] \zeta_J^{BM_2} \quad (19)$$

$$\langle b_{1t}^{\text{ew}}\zeta_J^{BM_2}\rangle_{M_1} = -\frac{3}{2} \left[ C_{10} + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_9}{N_c} \right] \zeta_J^{BM_2}$$

$$\langle b_{2u}\zeta_J^{BM_2}\rangle_{M_1} = \left[ C_2 + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_1}{N_c} \right] \zeta_J^{BM_2}$$

$$\langle b_{2t}^{\text{ew}}\zeta_J^{BM_2}\rangle_{M_1} = -\frac{3}{2} \left[ C_9 + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_{10}}{N_c} \right] \zeta_J^{BM_2}$$

$$\langle b_{3t}^{\text{ew}}\zeta_J^{BM_2}\rangle_{M_1} = -\frac{3}{2} \left[ C_7 + (1-\langle x^{-1}\rangle_{M_1}) \frac{C_8}{N_c} \right] \zeta_J^{BM_2}$$

$$\langle b_{4t}^p\zeta_J^{BM_2}\rangle_{M_1} = - \left[ C_4 + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_3}{N_c} \right] \zeta_J^{BM_2}$$

$$\langle b_{4t}^{\text{ew}}\zeta_J^{BM_2}\rangle_{M_1} = -\frac{1}{2} \left[ C_{10} + (1+\langle x^{-1}\rangle_{M_1}) \frac{C_9}{N_c} \right] \zeta_J^{BM_2}$$

and the remaining  $\langle b_i\zeta_J^{BM_2}\rangle_{M_1}$  vanish at this order. Here  $\langle x^{-1}\rangle_M \equiv \int_0^1 dx x^{-1} \phi_M(x)$ .

Finally, we consider also the case of SU(3) symmetry, where all the amplitude parameters for  $B$  decays to any two mesons  $M_1, M_2$  belonging to the same SU(3) multiplets are equal

$$G_{PP} = G, \quad G_{VV} = \tilde{G}, \quad G_{PV}^P = G^P, \quad G_{PV}^V = G^V. \quad (20)$$

where again  $G = \{T, C, P, EW^T, EW^C, EW^P\}$  denotes any of the graphical amplitudes. The amplitudes with

no subscripts are the amplitudes used in Refs. [3, 4, 5] and all results presented here reproduce the well known relations of these papers.

There is another important simplification which takes place for the EW penguin amplitudes in limit of flavor SU(3) symmetry. Assuming the dominance of the EWP operators  $Q_{9,10}$  one can show that the EWP graphical amplitudes are directly related to the tree level amplitudes [5, 17, 18]. With the normalization adopted here, these relations read

$$\begin{aligned} EW(\bar{K}^0\pi^-) + \sqrt{2}EW(K^-\pi^0) &= \frac{3}{2}\kappa_+(T+C) \\ EW(K^-\pi^+) + EW(\bar{K}^0\pi^-) &= \\ \frac{3}{4}\kappa_-(C-T) + \frac{3}{4}\kappa_+(C+T), \end{aligned} \quad (21)$$

where  $EW(M_1M_2) \equiv EW^T(M_1M_2) + EW^C(M_1M_2) + EW^P(M_1M_2)$  and we have defined

$$\kappa_{\pm} \equiv \frac{C_9 \pm C_{10}}{C_1 \pm C_2} \quad (22)$$

This gives, to all orders in matching

$$\begin{aligned} c_{1t}^{\text{ew}}(\omega_j) &= \frac{3}{4}\kappa_-[c_{1u}(\omega_j) - c_{2u}(\omega_j)] \\ &\quad - \frac{3}{4}\kappa_+[c_{1u}(\omega_j) + c_{2u}(\omega_j)] \\ b_{1t}^{\text{ew}}(\omega_j) &= \frac{3}{4}\kappa_-[b_{1u}(\omega_j) - b_{2u}(\omega_j)] \\ &\quad - \frac{3}{4}\kappa_+[b_{1u}(\omega_j) + b_{2u}(\omega_j)] \\ c_{2t}^{\text{ew}}(\omega_j) &= -\frac{3}{4}\kappa_-[c_{1u}(\omega_j) - c_{2u}(\omega_j)] \\ &\quad - \frac{3}{4}\kappa_+[c_{1u}(\omega_j) + c_{2u}(\omega_j)] \\ b_{2t}^{\text{ew}}(\omega_j) &= -\frac{3}{4}\kappa_-[b_{1u}(\omega_j) - b_{2u}(\omega_j)] \\ &\quad - \frac{3}{4}\kappa_+[b_{1u}(\omega_j) + b_{2u}(\omega_j)] \end{aligned} \quad (23)$$

These relations are satisfied by the tree level Wilson coefficients in Eqs. (6) and (7), but they are correct to all

orders in  $\alpha_s(m_b)$ .

A similar relation can be written for  $c_{4t}^{\text{ew}}$  provided that one makes the approximation  $\kappa_+ \simeq \kappa_- \equiv \kappa$ . Numerically this is well satisfied, with  $\kappa_+(m_b) = -8.75 \times 10^{-3}$  and  $\kappa_-(m_b) = -9.31 \times 10^{-3}$  [14]. Working in this approximation, one has [5]

$$EW_{M_1M_2}^P = \kappa P_{M_1M_2}^u \quad (24)$$

which gives another relation among SCET Wilson coefficients

$$c_{4t}^{\text{ew}}(\omega_i) + \frac{1}{3}c_{1t}^{\text{ew}}(\omega_i) = \kappa c_{4u}(\omega_i) \quad (25)$$

In conclusion, the results of our paper establish the connection between the graphical amplitudes, often used to parameterize nonleptonic B decay amplitudes, and the matrix elements of SCET operators. This allows an easier interpretation of phenomenological fits to data such as those performed in Refs. [19] in terms of nonperturbative SCET parameters. Such an analysis could help to constrain or extract the SCET parameters  $\zeta^{BM}$  and  $\zeta_J^{BM}$  in a way similar to the  $B \rightarrow \pi\pi$  study done in Ref. [13]. Although the analysis here was limited to the leading order in the  $1/m_b$  expansion, in principle it could be extended using the SCET expansion to include also power corrections. Finally, the model-independent relations Eqs. (23) fix the dominant electroweak penguin amplitudes and provide useful checks of perturbative matching computations in SCET.

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